



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 11

WISKUNDE P2

MODEL 2013

MEMORANDUM

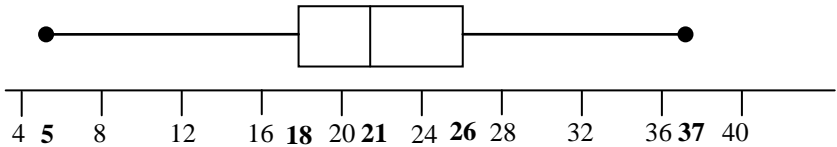
PUNTE: 150

Hierdie memorandum bestaan uit 13 bladsye.

NOTA:

- Indien ‘n kandidaat ‘n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- Indien ‘n kandidaat ‘n poging om ‘n vraag te beantwoord gekanselleer het en die vraag nie weer gedoen het nie, sien die gekanselleerde poging na.
- Konstante akkuraatheid is van toepassing in **ALLE** aspekte van die merk memorandum.
- Aanvaarding van antwoorde/waardes om ‘n probleem op te los, is **ONAANVAARBAAR**.

VRAAG 1

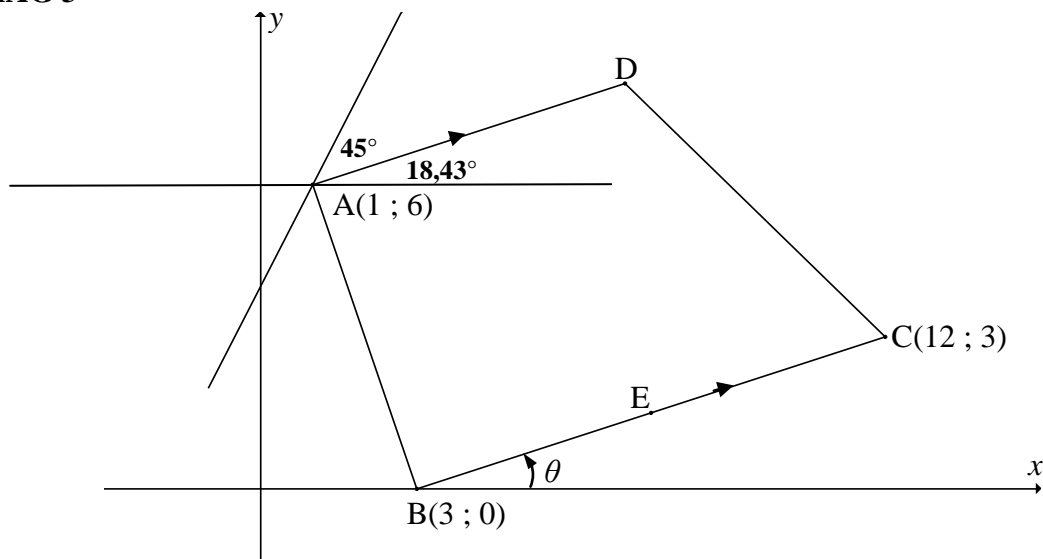
1.1	$\text{Gemiddelde} = \frac{\sum_{i=1}^n x_i}{n} = \frac{408}{19} = 21,47$	$\checkmark \frac{408}{19}$ \checkmark antwoord (2)																				
1.2	Standaardafwyking = 7,81	$\checkmark \checkmark$ antwoord (2)																				
1.3	Limiete vir een standaardafwyking is $(\bar{x} - 1\sigma; \bar{x} + 1\sigma)$ $= (21,47 - 7,81; 21,47 + 7,81) = (13,66 ; 29,28)$ \therefore 13 persone lê binne een standaardafwyking vanaf die gemiddelde	\checkmark interval \checkmark 13 persone (2)																				
1.4	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">12</td> <td style="padding: 2px 10px;">13</td> <td style="padding: 2px 10px;">15</td> <td style="padding: 2px 10px; background-color: #cccccc;">18</td> <td style="padding: 2px 10px;">18</td> <td style="padding: 2px 10px;">18</td> <td style="padding: 2px 10px;">19</td> <td style="padding: 2px 10px;">20</td> <td style="padding: 2px 10px; background-color: #cccccc;">21</td> </tr> <tr> <td style="padding: 2px 10px;">21</td> <td style="padding: 2px 10px;">22</td> <td style="padding: 2px 10px;">23</td> <td style="padding: 2px 10px;">23</td> <td style="padding: 2px 10px; background-color: #cccccc;">26</td> <td style="padding: 2px 10px;">29</td> <td style="padding: 2px 10px;">33</td> <td style="padding: 2px 10px;">35</td> <td style="padding: 2px 10px;">37</td> <td></td> </tr> </table> IKW = 26 – 18 = 8	5	12	13	15	18	18	18	19	20	21	21	22	23	23	26	29	33	35	37		$\checkmark Q_1 = 18$ $\checkmark Q_3 = 26$ \checkmark IKW = 8 (3)
5	12	13	15	18	18	18	19	20	21													
21	22	23	23	26	29	33	35	37														
1.5		$\checkmark \checkmark$ mond \checkmark snorre (3)																				
1.6	Daar is ‘n merkbare verskil tussen die laagste waarde (5) en die volgende laagste waarde (12), terwyl die verskil tussen alle ander waardes op die meeste 3 is. \therefore 5 is ‘n uitskieter	\checkmark rede \checkmark 5 ‘n uitskieter (2) [14]																				

VRAAG 2

<p>2.1</p>	<table border="1"> <thead> <tr> <th>Klas</th> <th>Frekwensie</th> <th>Kumulatiewe frekwensie</th> </tr> </thead> <tbody> <tr> <td>$0 \leq m < 2$</td> <td>7</td> <td>7</td> </tr> <tr> <td>$2 \leq m < 4$</td> <td>15</td> <td>22</td> </tr> <tr> <td>$4 \leq m < 6$</td> <td>26</td> <td>48</td> </tr> <tr> <td>$6 \leq m < 8$</td> <td>29</td> <td>77</td> </tr> <tr> <td>$8 \leq m < 10$</td> <td>36</td> <td>113</td> </tr> <tr> <td>$10 \leq m < 12$</td> <td>31</td> <td>144</td> </tr> <tr> <td>$12 \leq m < 14$</td> <td>14</td> <td>158</td> </tr> <tr> <td>$14 \leq m < 16$</td> <td>2</td> <td>160</td> </tr> </tbody> </table>	Klas	Frekwensie	Kumulatiewe frekwensie	$0 \leq m < 2$	7	7	$2 \leq m < 4$	15	22	$4 \leq m < 6$	26	48	$6 \leq m < 8$	29	77	$8 \leq m < 10$	36	113	$10 \leq m < 12$	31	144	$12 \leq m < 14$	14	158	$14 \leq m < 16$	2	160	<p>✓ eerste drie kumulatiewe frekwensies korrek ✓ res korrek (totaal = 160) (2)</p>
Klas	Frekwensie	Kumulatiewe frekwensie																											
$0 \leq m < 2$	7	7																											
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$12 \leq m < 14$	14	158																											
$14 \leq m < 16$	2	160																											
<p>2.2</p>		<p>✓ geanker by 0 ✓ plot kumulatiewe frekwensie teen die boonste limietwaardes ✓ gladde grafiek-kurwe (3)</p>																											

2.3	Die mediaan vir die data is ongeveer 8 boodskappe.	✓Mediaan (1)
2.4	Ongeveer 130 leerders het 11 of minder boodskappe gestuur. Dus het 30 leerders meer as 11 boodskappe gestuur. $\frac{30}{160} \times 100\% = 18,75\%$	✓30 leerders ✓antwoord (2)
2.5	Skeef na links of negatief skeef	✓antwoord (1)
		[9]

VRAAG 3



3.1	$E\left(\frac{3+12}{2}; \frac{0+3}{2}\right)$ $= \left(7\frac{1}{2}; 1\frac{1}{2}\right)$	✓ substitusie in middelpunt- formule ✓ antwoord (2)
3.2	$m_{BC} = \frac{3-0}{12-3}$ $= \frac{1}{3}$	✓ substitusie in gradiëntformule ✓ antwoord (2)
3.3	$\tan \theta = m_{BC} = \frac{1}{3}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18,43^\circ$	✓ $\tan \theta = m_{BC}$ ✓ antwoord (2)
3.4	$m_{AD} = m_{BC} = \frac{1}{3} \quad AD \parallel BC, \text{ gradiënte gelyk}$ $m_{AB} = \frac{6-0}{1-3} = -3$ $\therefore m_{AD} \times m_{AB} = \frac{1}{3} \times -3 = -1$ $\therefore AD \perp AB$	✓ $m_{AD} = \frac{1}{3}$ ✓ $m_{AB} = -3$ ✓ $m_{AD} \times m_{AB} = -1$ (3)
3.5	inklinasie van nuwe lyn = $45^\circ + 18,43^\circ = 63,43^\circ$ $\therefore \tan 63,43^\circ = 2 = m_{lyn}$ $\therefore y - 6 = 2(x - 1)$ $y = 2x + 4$	✓ $18,43^\circ$ ✓ $63,43^\circ$ ✓ $m = 2$ ✓ subst van (1 ; 6) ✓ vergelyking (5) [14]

VRAAG 4

4.1	$m_{QP} = m_{OS} = 6$ $QP \parallel OS$, gelyke gradiënte $y - 17 = 6(x + 3)$ $y = 6x + 35$	$\checkmark m_{QP} = 6$ \checkmark subst $(-3 ; 17)$ in formule \checkmark vergelyking (3)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) = 5$ $\therefore Q(-5 ; 5)$	OF $y = 6(-5) + 35 = 5$ \checkmark stel vergelyking op $\checkmark x = -5$ $\checkmark y = 5$ \checkmark koördinate van Q (4)
4.3	$OQ^2 = (-5 - 0)^2 + (5 - 0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ eenhede	\checkmark substitusie in afstandsformule $\checkmark 5\sqrt{2}$ (2)
4.4	$m_{OS} = 6$ \therefore inklinasie van OS is $\tan^{-1}(6) = 80,54^\circ$ $m_{OQ} = -1$ \therefore inklinasie van QO is $180^\circ - \tan^{-1}(1) = 135^\circ$ $\alpha = 135^\circ - 80,54\dots^\circ$ $= 54,46^\circ$	$\checkmark 80,54^\circ$ $\checkmark 135^\circ$ $\checkmark 54,46^\circ$ (3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS \cdot OQ \cdot \cos \alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50} \cdot \cos 54,46^\circ)$ $QS = 9,90$ eenhede	\checkmark korrekte gebruik van cos-reël \checkmark substitusie in formule $\checkmark 9,90$ (3) [15]

VRAAG 5

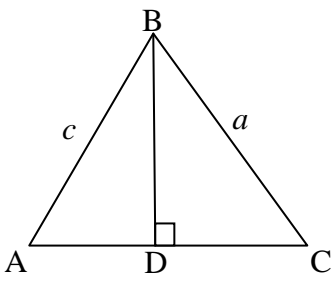
5.1.1	$\cos \alpha = -\frac{5}{13}$	$\checkmark -\frac{5}{13}$ (1)
5.1.2	$(-5)^2 + b^2 = 13^2$ $b^2 = 169 - 25 = 144$ $b = 12$ $\tan (180^\circ - \alpha)$ $= -\tan \alpha$ $= -\left(-\frac{12}{5}\right)$ $= \frac{12}{5}$	$\checkmark b = 12$ $\checkmark -\tan \alpha$ $\checkmark \frac{12}{5}$ (3)
5.2.1	$\frac{\sin(\theta - 360^\circ) \sin(90^\circ - \theta) \tan(-\theta)}{\cos(90^\circ + \theta)}$ $= \frac{\sin \theta \cos \theta (-\tan \theta)}{-\sin \theta}$ $= -\cos \theta \left(-\frac{\sin \theta}{\cos \theta} \right)$ $= \sin \theta$	$\checkmark \checkmark \checkmark$ reduksies $\checkmark \tan \theta = \frac{\sin \theta}{\cos \theta}$ $\checkmark \sin \theta$ (5)
5.2.2	Van 5.2.1: $\sin \theta = 0,5$ Verwysings $\angle = 30^\circ$ $\therefore \theta = 30^\circ$ of $\theta = 150^\circ$	$\checkmark \sin \theta = 0,5$ $\checkmark 30^\circ$ $\checkmark 150^\circ$ (3)

<p>5.3.1</p>	$LK = \frac{8}{\sin^2 A} - \frac{4}{1 + \cos A}$ $= \frac{8}{1 - \cos^2 A} - \frac{4}{1 + \cos A}$ $= \frac{8}{(1 - \cos A)(1 + \cos A)} - \frac{4}{1 + \cos A}$ $= \frac{8 - 4(1 - \cos A)}{(1 - \cos A)(1 + \cos A)}$ $= \frac{8 - 4 + 4 \cos A}{(1 - \cos A)(1 + \cos A)}$ $= \frac{4(1 + \cos A)}{(1 - \cos A)(1 + \cos A)}$ $= \frac{4}{1 - \cos A} = RK$	<p>✓ $\sin^2 A = 1 - \cos^2 A$</p> <p>✓ faktorisering</p> <p>✓ optelling</p> <p>✓ vereenvoudiging</p> <p>✓ faktorisering</p> <p>(5)</p>
<p>5.3.2</p>	<p>Identiteit is ongedefinieer as $\sin^2 A = 0$. Dit is as $\sin A = 0$ of $\cos A = \pm 1$ $\therefore A = 0^\circ$ of $A = 180^\circ$ of $A = 360^\circ$.</p>	<p>✓✓✓ elke waarde</p> <p>(3)</p>
<p>5.4</p>	$8 \cos^2 x - 2 \cos x - 1 = 0$ $(4 \cos x + 1)(2 \cos x - 1) = 0$ $\cos x = -\frac{1}{4} \text{ of } \cos x = \frac{1}{2}$ <p>$\therefore x = 104,48^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ of $x = 60^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$</p> <p>$x = 255,52^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $x = 300^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$</p>	<p>✓ faktorisering</p> <p>✓ waardes van $\cos x$</p> <p>✓ $104,48^\circ$ of $255,52^\circ$</p> <p>✓ 60° of 300°</p> <p>✓ $+ 360^\circ \cdot k$</p> <p>✓ $k \in \mathbb{Z}$</p> <p>(6)</p> <p>[26]</p>

VRAAG 6

6.1	$p = -45^\circ$ $q = -1$	✓ waarde van p ✓ waarde van q (2)
6.2	B(157,5° ; - 0,38)	✓ waarde van x ✓ waarde van y (2)
6.3	$f(x) < g(x)$ as $-180^\circ \leq x < -22,5^\circ$ of $157,5^\circ < x \leq 180^\circ$	✓ $-180^\circ \leq x < -22,5^\circ$ ✓ $157,5^\circ < x \leq 180^\circ$ (2)
6.4.1	$h(x) = \cos(x - 45^\circ + 30^\circ)$ $= \cos(x - 15^\circ)$	✓ $+ 30^\circ$ ✓ eenvoudigste vorm (2)
6.4.2	$x = -135^\circ - 30^\circ = -165^\circ$	✓ -165° (1) [9]

VRAAG 7

7.1	<p>Trek $BD \perp AC$ In $\triangle ABD$: $\sin A = \frac{BD}{c} \therefore BD = c \cdot \sin A$</p> <p>In $\triangle CBD$: $\sin C = \frac{BD}{a} \therefore BD = a \cdot \sin C$</p> <p>$\therefore c \cdot \sin A = a \cdot \sin C$ $\therefore \frac{\sin A}{a} = \frac{\sin C}{c}$</p>	 <p>✓ konstruksie ✓ $\sin A$ ✓ maak BD die onderwerp ✓ $\sin C$ ✓ $c \cdot \sin A = a \cdot \sin C$ (5)</p>
7.2.1	$\frac{\sin R}{r} = \frac{\sin P}{p}$ $\frac{\sin R}{27,2} = \frac{\sin 132^\circ}{73,2}$ $\sin R = \frac{27,2 \times \sin 132^\circ}{73,2}$ $= 0,276\dots$ $\hat{R} = 16,03^\circ$	<p>✓ substitusie in korrekte formule ✓ maak $\sin R$ die onderwerp ✓ $16,03^\circ$ (3)</p>

7.2.2	$\hat{Q} = 180^\circ - 132^\circ - 16,03^\circ = 31,97^\circ$ <p>oppervlakte van PQR = $\frac{1}{2} pr \cdot \sin Q$</p> $= \frac{1}{2} (73,2)(27,2) \cdot \sin 31,97^\circ$ $= 527,10 \text{ cm}^2$	$\checkmark \hat{Q} = 31,97^\circ$ \checkmark substitusie in korrekte formule $\checkmark 527,1$ (3)
7.3.1	$P\hat{S}Q = 180^\circ - (a + b)$ <p>In ΔPSQ:</p> $\frac{SQ}{\sin P} = \frac{PQ}{\sin P\hat{S}Q}$ $\frac{SQ}{\sin a} = \frac{h}{\sin[180^\circ - (a + b)]}$ $\frac{SQ}{\sin a} = \frac{h}{\sin(a + b)}$ $SQ = \frac{h \sin a}{\sin(a + b)}$	\checkmark $P\hat{S}Q = 180^\circ - (a + b)$ $\checkmark \sin[180^\circ - (a + b)] = \sin(a + b)$ \checkmark maak SQ die onderwerp (3)
7.3.2	$S\hat{Q}R = 90^\circ - b$ <p>In ΔRSQ:</p> $\frac{RS}{SQ} = \sin S\hat{Q}R$ $RS = SQ \cdot \sin(90^\circ - b)$ $= \frac{h \sin a}{\sin(a + b)} \cdot \cos b$ $= \frac{h \sin a \cdot \cos b}{\sin(a + b)}$	$\checkmark S\hat{Q}R = 90^\circ - b$ \checkmark gebruik sin-verhouding korrek $\checkmark \sin(90^\circ - b) = \cos b$ (3) [17]

VRAAG8

	<p>Volume van hemisfeer</p> $= \frac{1}{2} \left[\frac{4}{3} \pi r^3 \right]$ $= \frac{2}{3} \pi (3)^3$ $= 18\pi \text{ cm}^3$ <p>Volume van keëlvormige gat</p> $= \frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi (1,5)^2 \left(\frac{8}{9}\right)$ $= \frac{2}{3} \pi \text{ cm}^3$ $\therefore \frac{\text{volume van metaal A}}{\text{volume van metaal B}} = \frac{17\frac{1}{3}\pi}{\frac{2}{3}\pi} = \frac{26}{1}$ <p>Verhouding van volume van metaal A : Volume van metaal B</p> $= 26 : 1$	<p>✓ substitusie in korrekte formule</p> <p>✓ 18 π</p> <p>✓ substitusie in korrekte formule</p> <p>✓ $\frac{2}{3}\pi$</p> <p>✓ $17\frac{1}{3}\pi$</p> <p>✓ verhouding 26 : 1</p> <p style="text-align: right;">(6) [6]</p>
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VRAAG9

9.1	... halveer die koord.	✓ antwoord (1)
9.2.1	<p>OE = 10 cm ... O middelpunt van DE</p> <p>OC = OE – CE</p> <p>= 10 – 2</p> <p>= 8 cm</p>	<p>✓ OE = 10</p> <p>✓ OC = 8</p> <p style="text-align: right;">(2)</p>
9.2.2	<p>In ΔCOQ:</p> <p>QC² = OQ² – OC² ... Stelling van Pythagoras</p> <p>= (10)² – (8)²</p> <p>= 36</p> <p>QC = 6 cm</p> <p>∴ PQ = 2QC ... loodlyn vanaf middelpunt van sirkel na koord, halveer koord</p> <p>PQ = 12 cm</p>	<p>✓ Gebruik stelling van Pythagoras</p> <p>✓ QC = 6</p> <p>✓ PQ = 12 (S)</p> <p>✓ rede</p> <p style="text-align: right;">(4) [7]</p>

VRAAG 10

<p>10.1</p>	<div style="text-align: center;"> </div> <p>Konstruksie: Verleng DO na E. Bewys: In $\triangle OBD$: $O\hat{B}D = O\hat{D}B$... $OD = OB = r$ $E\hat{O}B = 2 \times O\hat{D}B$... buitehoek van driehoek In $\triangle AOD$: $O\hat{A}D = O\hat{D}A$... $OA = OD = r$ $E\hat{O}A = 2 \times O\hat{D}A$... buitehoek van driehoek $A\hat{O}B = E\hat{O}B + E\hat{O}A$ $= 2 \times O\hat{D}B + 2 \times O\hat{D}A$ $= 2(O\hat{D}B + O\hat{D}A)$ $= 2A\hat{D}B$</p>	<p>✓ konstruksie</p> <p>✓ $O\hat{B}D = O\hat{D}B$ ✓ $E\hat{O}B = 2 \times O\hat{D}B$ (S/R)</p> <p>✓ $E\hat{O}A = 2 \times O\hat{D}A$ (S/R) ✓ $A\hat{O}B = E\hat{O}B + E\hat{O}A$</p> <p style="text-align: right;">(5)</p>
<p>10.2.1(a)</p>	<p>$\hat{M} = 76^\circ$... middelpunts$\angle = 2(\text{omtreks}\angle)$</p>	<p>✓ 76° ✓ rede</p> <p style="text-align: right;">(2)</p>
<p>10.2.1(b)</p>	<p>$\hat{T}_2 = 38^\circ$... buite\angle van koordevierhoek KTAB</p>	<p>✓ 38° ✓ rede</p> <p style="text-align: right;">(2)</p>
<p>10.2.1(c)</p>	<p>$\hat{C} = 38^\circ$... buite\angle van koordevierhoek of \angle^e in dieselfde segment</p>	<p>✓ 38° ✓ rede</p> <p style="text-align: right;">(2)</p>
<p>10.2.1(d)</p>	<p>$\hat{C}\hat{A}N = \hat{C} = 38^\circ$... $NA = NC$ $\hat{K}_4 = 38^\circ$... buite\angle van koordevierhoek CATK</p>	<p>✓ $\hat{C}\hat{A}N = 38^\circ$ (S/R) ✓ $\hat{K}_4 = 38^\circ$</p> <p style="text-align: right;">(2)</p>
<p>10.2.2</p>	<p>$\therefore \hat{K}_4 = \hat{T}_2$ $\therefore NK = NT$... basis\angle^e gelyk</p>	<p>✓ stelling ✓ rede</p> <p style="text-align: right;">(2)</p>
<p>10.2.3</p>	<p>$\hat{N} = 180^\circ - (38^\circ + 38^\circ)$... \angle^e van $\triangle KNT$ $= 104^\circ$ $\hat{N} + \hat{K}\hat{M}A = 104^\circ + 76^\circ = 180^\circ$ \therefore AMKN is 'n koordevierhoek ... teenoorstaande $\angle^e = 180^\circ$</p>	<p>✓ $\hat{N} = 104^\circ$ (S/R) ✓ $\hat{N} + \hat{K}\hat{M}A = 180^\circ$ ✓ rede</p> <p style="text-align: right;">(3) [18]</p>

VRAAG 11

11.1 gelyk aan die hoek onderspan deur dieselfde koord in die (alternatiewe) teenoorstaande segment	✓ teenoorstaande segment (1)
11.2.1	$\hat{A}_1 = \hat{C}_2 = x$...raaklyn-koord-stelling $\hat{C}_2 = \hat{G}_2 = x$...raaklyn-koord-stelling $\therefore \hat{A}_1 = \hat{G}_2 = x$ $\therefore BCG \parallel EA$...verwissellende $\sphericalangle^e =$	✓ $\hat{A}_1 = \hat{C}_2 = x$ ✓ rede ✓ $\hat{C}_2 = \hat{G}_2 = x$ ✓ rede ✓ gevolgtrekking met rede (5)
11.2.2	$\hat{E}_1 = \hat{C}_3 = y$...verwissellende \sphericalangle^e ; $BG \parallel EA$ $\hat{F}_1 = \hat{C}_3 = y$...buite \sphericalangle van koordevierhoekCDFG $\therefore \hat{E}_1 = \hat{F}_1 = y$ $\therefore EA$ is a tangent ...omgekeerde raaklyn-koord-stelling	✓ $\hat{E}_1 = \hat{C}_3 = y$ (S/R) ✓ $\hat{F}_1 = \hat{C}_3 = y$ (S) ✓ rede ✓ $\hat{E}_1 = \hat{F}_1 = y$ ✓ rede (5)
11.2.3	$\hat{B} = \hat{C}\hat{A}\hat{E}$...raaklyn-koord stelling $\hat{C}_1 = \hat{C}\hat{A}\hat{E}$...verwissellende \sphericalangle^e ; $BG \parallel EA$ $\hat{C}_1 = \hat{B}$ $\therefore AB = AC$...basis $\sphericalangle^e =$	✓ $\hat{C}\hat{A}\hat{E} = \hat{B}$ ✓ rede ✓ $\hat{C}_1 = \hat{C}\hat{A}\hat{E}$ (S/R) ✓ rede (4) [15]

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